Conventions and Notation

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0.1 Scope

This page fixes, once and for all, the units, metric, and index conventions used throughout the electrodynamics notes. Every other page in this course assumes these conventions; when a formula elsewhere looks unfamiliar, check here first.

1 Units

Unless otherwise stated, SI units are used throughout. The vacuum constants satisfy

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.$$

2 Metric

The Minkowski metric is

$$\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1),$$

so that

$$x^2 = \eta_{\mu\nu} x^\mu x^\nu = -(x^0)^2 + \mathbf{x}^2.$$

Greek indices run from 0 to 3, while Latin indices run from 1 to 3. Repeated indices are summed.

3 Coordinates

We write

$$x^\mu = (ct, \mathbf{x}), \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}.$$

The d'Alembertian is

$$\Box = \partial_\mu \partial^\mu.$$

For two spacetime points,

$$\Delta x^\mu = x^\mu - y^\mu,$$

with

$$(\Delta x)^2 = \eta_{\mu\nu} \Delta x^\mu \Delta x^\nu.$$

4 Electromagnetic field

The electromagnetic tensor is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Its components are

$$F^{0i} = \frac{E^i}{c}, \qquad F^{ij} = \epsilon^{ijk} B_k.$$

The four-potential is

$$A^\mu = \left(\frac{\phi}{c}, \mathbf{A}\right).$$

The four-current is

$$J^\mu = (c\rho, \mathbf{J}).$$

5 Maxwell equations

The inhomogeneous equations are

$$\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu.$$

The homogeneous equations are

$$\partial_{[\alpha} F_{\beta\gamma]} = 0.$$

In the Lorenz gauge,

$$\partial_\mu A^\mu = 0,$$

the field equations reduce to

$$\Box A^\mu = -\mu_0 J^\mu.$$

6 Energy-momentum tensor

$$T^{\mu\nu} = \frac{1}{\mu_0}\left(F^{\mu\alpha}F^{\nu}{}_{\alpha} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right),$$

with

$$T^{00} = u = \frac{1}{2\mu_0}\left(\frac{E^2}{c^2} + B^2\right), \qquad T^{0i} = \frac{S^i}{c}, \qquad \mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}.$$

7 Particle kinematics

The worldline is $y^\mu(\tau)$, and the proper time satisfies

$$c^2 \mathrm{d}\tau^2 = -ds^2.$$

The four-velocity is

$$u^\mu = \frac{\mathrm{d}y^\mu}{c\,\mathrm{d}\tau} = \gamma\left(1, \frac{\mathbf{v}}{c}\right),$$

with

$$u^\mu u_\mu = -1.$$

The four-acceleration is

$$a^\mu = \frac{\mathrm{d}u^\mu}{\mathrm{d}\tau},$$

and obeys

$$u_\mu a^\mu = 0.$$

A dot is reserved for the derivative with respect to coordinate time t, e.g. $\dot{\mathbf{v}} = \mathrm{d}\mathbf{v}/\mathrm{d}t$; the proper-time derivative above is always written out or given its own symbol (a^μ), never as $\dot{u}^\mu$.

8 Useful identities

The retarded Green function satisfies

$$\Box G_R(x - x') = \delta^{(4)}(x - x').$$

The point-particle current is

$$J^\mu(x) = cq \int u^\mu(\tau)\delta^{(4)}(x - y(\tau))\,\mathrm{d}\tau.$$

Useful distribution identity:

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|}.$$

Feel free to create issues, ask questions, or suggest improvements in the GitHub repository.